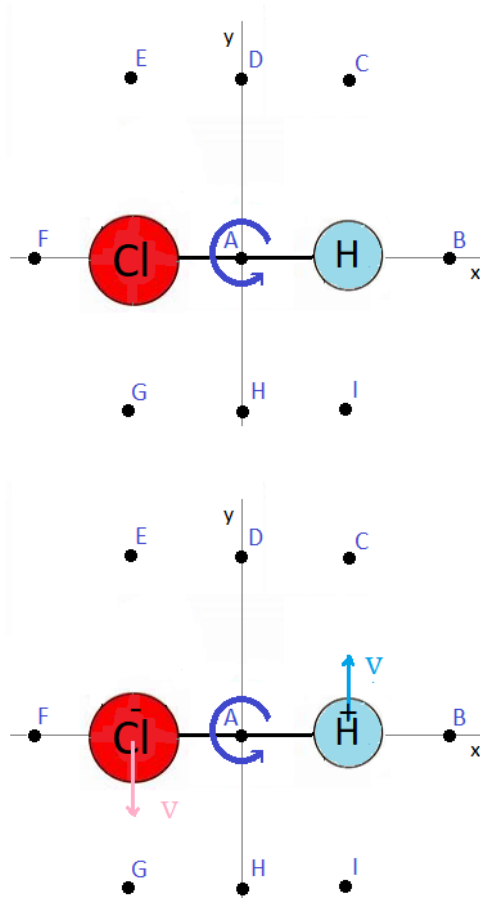


## Homework 10: BF(ields)F

Due 6/8

**Problem 1.** Remember the HCl molecule? Recall it has a bond length of 127pm. Cl tends to 'borrow' H's electron, giving it an 'effective' charge of  $-1e$ , and leaving H with an 'effective' charge of  $+1e$ , where  $e = 1.6 \times 10^{-19} \text{C}$ . At room temperatures, it will rotate with a frequency of around  $f = 10^{12} \text{Hz}$ . Say it's rotating about the z axis with this frequency. Then,



(a) Calculate the speed of each atom (remember PHY 141)?

So,

$$v = \omega r = 2\pi f r = 2\pi(10^{12})(63.5 \times 10^{-12}) = 400 \text{m/s}$$

(b) Calculate the magnetic field at point A = (0pm,0pm).

This is:

$$\begin{aligned}
\mathbf{B} &= \mathbf{B}_{Cl} + \mathbf{B}_H \\
&= \frac{k_m |q| v}{r^2} \sin \beta @ -RHR + \frac{k_m |q| v}{r^2} \sin \beta @ RHR \\
&= \frac{(10^{-7})e(400)}{(63.5 \times 10^{-12})^2} \sin 90^\circ @ (-\hat{\mathbf{k}}) + \frac{(10^{-7})e(400)}{(63.5 \times 10^{-12})^2} \sin 90^\circ \hat{\mathbf{k}} \\
&= 0
\end{aligned}$$

(c) Calculate the magnetic field at point B = (100pm, 0pm).

And,

$$\begin{aligned}
\mathbf{B} &= \mathbf{B}_{Cl} + \mathbf{B}_H \\
&= \frac{k_m |q| v}{r^2} \sin \beta @ -RHR + \frac{k_m |q| v}{r^2} \sin \beta @ RHR \\
&= \frac{(10^{-7})e(400)}{(163.5 \times 10^{-12})^2} \sin 90^\circ @ (-\hat{\mathbf{k}}) + \frac{(10^{-7})e(400)}{(36.5 \times 10^{-12})^2} \sin 90^\circ (-\hat{\mathbf{k}}) \\
&= -5\text{mT } \hat{\mathbf{k}}
\end{aligned}$$

(d) Calculate the magnetic field at point C = (63.5pm, 100pm).

And,

$$\begin{aligned}
\mathbf{B} &= \mathbf{B}_{Cl} + \mathbf{B}_H \\
&= \frac{k_m |q| v}{r^2} \sin \beta @ -RHR + \frac{k_m |q| v}{r^2} \sin \beta @ RHR \\
&= \frac{(10^{-7})e(400)}{(127 \times 10^{-12})^2 + (100 \times 10^{-12})^2} \cdot \frac{100 \times 10^{-12}}{\sqrt{(127 \times 10^{-12})^2 + (100 \times 10^{-12})^2}} @ (-\hat{\mathbf{k}}) + \frac{(10^{-7})e(400)}{(100 \times 10^{-12})^2} \sin 0^\circ @ RHR \\
&= -152\mu\text{T } \hat{\mathbf{k}}
\end{aligned}$$

(e) Calculate the magnetic field at point D = (0pm, 100pm).

And,

$$\begin{aligned}
\mathbf{B} &= \mathbf{B}_{Cl} + \mathbf{B}_H \\
&= \frac{k_m |q| v}{r^2} \sin \beta @ -RHR + \frac{k_m |q| v}{r^2} \sin \beta @ RHR \\
&= \frac{(10^{-7})e(400)}{(36.5 \times 10^{-12})^2 + (100 \times 10^{-12})^2} \cdot \frac{100 \times 10^{-12}}{\sqrt{(36.5 \times 10^{-12})^2 + (100 \times 10^{-12})^2}} @ (-\hat{\mathbf{k}}) \\
&\quad + \frac{(10^{-7})e(400)}{(36.5 \times 10^{-12})^2 + (100 \times 10^{-12})^2} \cdot \frac{100 \times 10^{-12}}{\sqrt{(36.5 \times 10^{-12})^2 + (100 \times 10^{-12})^2}} @ (\hat{\mathbf{k}}) \\
&= 0
\end{aligned}$$

(f) Calculate the magnetic field at point E = (-100pm, 63.5pm).

And,

$$\begin{aligned}
 \mathbf{B} &= \mathbf{B}_{Cl} + \mathbf{B}_H \\
 &= \frac{k_m |q| v}{r^2} \sin \beta @ -RHR + \frac{k_m |q| v}{r^2} \sin \beta @ RHR \\
 &= \frac{(10^{-7})e(400)}{(100 \times 10^{-12})^2} \sin 0^\circ @ RHR + \frac{(10^{-7})e(400)}{(127 \times 10^{-12})^2 + (100 \times 10^{-12})^2} \cdot \frac{100 \times 10^{-12}}{\sqrt{(127 \times 10^{-12})^2 + (100 \times 10^{-12})^2}} @ (\hat{\mathbf{k}}) \\
 &= 152 \mu\text{T } \hat{\mathbf{k}}
 \end{aligned}$$

(g) Calculate the magnetic field at point F = (-100pm, 0pm).

And,

$$\begin{aligned}
 \mathbf{B} &= \mathbf{B}_{Cl} + \mathbf{B}_H \\
 &= \frac{k_m |q| v}{r^2} \sin \beta @ -RHR + \frac{k_m |q| v}{r^2} \sin \beta @ RHR \\
 &= \frac{(10^{-7})e(400)}{(36.5 \times 10^{-12})^2} \sin 90^\circ (-\hat{\mathbf{k}}) + \frac{(10^{-7})e(400)}{(163.5 \times 10^{-12})^2} \sin 90^\circ @ (-\hat{\mathbf{k}}) \\
 &= 5 \text{mT } \hat{\mathbf{k}}
 \end{aligned}$$

(h) Calculate the magnetic field at point G = (-63.5pm, -100pm).

And,

$$\begin{aligned}
 \mathbf{B} &= \mathbf{B}_{Cl} + \mathbf{B}_H \\
 &= \frac{k_m |q| v}{r^2} \sin \beta @ -RHR + \frac{k_m |q| v}{r^2} \sin \beta @ RHR \\
 &= \frac{(10^{-7})e(400)}{(100 \times 10^{-12})^2} \sin 0^\circ @ RHR + \frac{(10^{-7})e(400)}{(127 \times 10^{-12})^2 + (100 \times 10^{-12})^2} \cdot \frac{100 \times 10^{-12}}{\sqrt{(127 \times 10^{-12})^2 + (100 \times 10^{-12})^2}} @ (\hat{\mathbf{k}}) \\
 &= 152 \mu\text{T } \hat{\mathbf{k}}
 \end{aligned}$$

(i) Calculate the magnetic field at point H = (0pm, -100pm).

And,

$$\begin{aligned}
\mathbf{B} &= \mathbf{B}_{Cl} + \mathbf{B}_H \\
&= \frac{k_m |q| v}{r^2} \sin \beta @ -RHR + \frac{k_m |q| v}{r^2} \sin \beta @ RHR \\
&= \frac{(10^{-7})e(400)}{(36.5 \times 10^{-12})^2 + (100 \times 10^{-12})^2} \cdot \frac{100 \times 10^{-12}}{\sqrt{(36.5 \times 10^{-12})^2 + (100 \times 10^{-12})^2}} @ (-\hat{\mathbf{k}}) \\
&\quad + \frac{(10^{-7})e(400)}{(36.5 \times 10^{-12})^2 + (100 \times 10^{-12})^2} \cdot \frac{100 \times 10^{-12}}{\sqrt{(36.5 \times 10^{-12})^2 + (100 \times 10^{-12})^2}} @ (\hat{\mathbf{k}}) \\
&= 0
\end{aligned}$$

(j) Calculate the magnetic field at point I = (63.5pm, -100pm).

And,

$$\begin{aligned}
\mathbf{B} &= \mathbf{B}_{Cl} + \mathbf{B}_H \\
&= \frac{k_m |q| v}{r^2} \sin \beta @ -RHR + \frac{k_m |q| v}{r^2} \sin \beta @ RHR \\
&= \frac{(10^{-7})e(400)}{(127 \times 10^{-12})^2 + (100 \times 10^{-12})^2} \cdot \frac{100 \times 10^{-12}}{\sqrt{(127 \times 10^{-12})^2 + (100 \times 10^{-12})^2}} @ (-\hat{\mathbf{k}}) + \frac{(10^{-7})e(400)}{(100 \times 10^{-12})^2} \sin 0^\circ @ RHR \\
&= -152 \mu\text{T } \hat{\mathbf{k}}
\end{aligned}$$

**Problem 2.** A lightning bolt carries a typical current of about 50kA. Approximating the current as following a straight line, infinitely long, what magnetic field would this current produce R = 100m away?

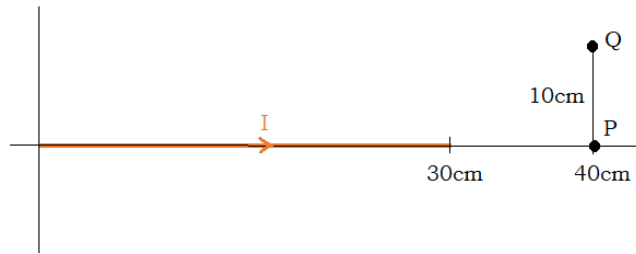


We can approximate the lightning bolt as a long straight wire. In that case,

$$\begin{aligned}
B &= \frac{\mu_0 I}{2\pi R} \\
&= \frac{(4\pi \times 10^{-7})(50 \times 10^3)}{2\pi(100)} \\
&= 1 \times 10^{-4} \text{ T} = 100 \mu\text{T}
\end{aligned}$$

The field would be about 10 times that of Earth's natural magnetic field.

**Problem 3.** Consider the 30cm long wire segment carrying current I = 5A.

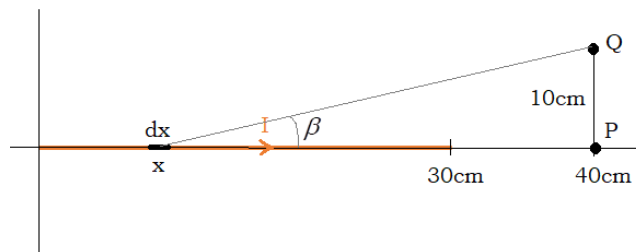


(a) What is the magnetic field at point P?

$B = 0$  at point P since the point is in line with the current.

(b) What is the magnetic field at point Q?

At point Q we have:

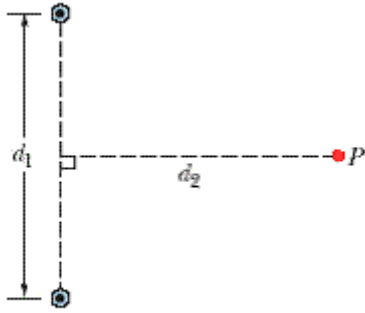


$$\begin{aligned}
 \mathbf{B} &= \int d\mathbf{B} \\
 &= \int \frac{k_m |dq| v}{r^2} \sin \beta @ RHR \\
 &= \int \frac{k_m I dx}{(0.40 - x)^2 + 0.10^2} \frac{0.10}{\sqrt{(0.40 - x)^2 + 0.10^2}} \hat{\mathbf{k}} \\
 &= \int_0^{0.30} \frac{0.10 k_m I dx}{[(0.40 - x)^2 + 0.10^2]^{3/2}} \hat{\mathbf{k}} \\
 &= 0.10 k_m I \cdot \frac{x - 0.40}{(0.10)^2 \sqrt{(0.40 - x)^2 + 0.10^2}} \bigg|_{x=0}^{x=0.30} \hat{\mathbf{k}} \\
 &= 1.3 \mu\text{T} \hat{\mathbf{k}}
 \end{aligned}$$

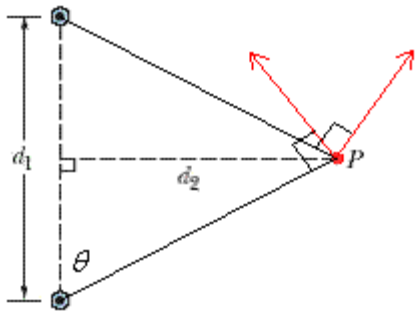
(c) Where, generally do you think the field would be largest?

Along the wire bisector.

**Problem 4.** The figure below shows two very long straight wires (in cross section) that each carry a current of 5 A directly out of the page. Distance  $d_1 = 3\text{m}$  and distance  $d_2 = 5\text{m}$ . What is the magnitude of the net magnetic field at point P, which lies on a perpendicular bisector to the wires?



We have:



Angle  $\theta$  that currents make with the point is:

$$\theta = \tan^{-1}\left(\frac{d_2}{d_1/2}\right) = \tan^{-1}\left(\frac{5}{1.5}\right) = 73.3^\circ$$

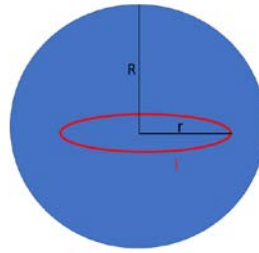
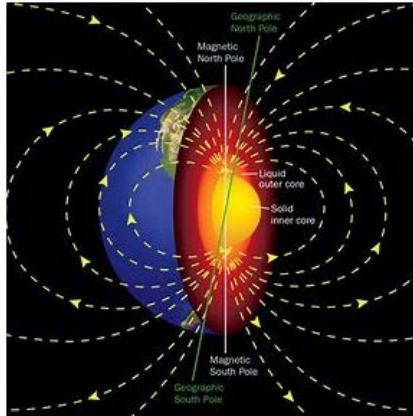
Magnetic field of each is:

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7})(5)}{2\pi \sqrt{(d_1/2)^2 + d_2^2}} = \frac{(4\pi \times 10^{-7})(5)}{2\pi \sqrt{(3/2)^2 + 5^2}} = 1.92 \times 10^{-7} \text{ T}$$

Their horizontal components cancel, and their vertical components add, to give a net value of:

$$\mathbf{B} = 2(1.92 \times 10^{-7} \text{ T}) \sin \theta = 3.67 \times 10^{-7} \text{ T}$$

**Problem 5.** Earth's magnetic field has a strength of around  $50 \mu\text{T}$  at the poles. And it is thought to be caused by currents circulating in the core. Approximate the current as a current loop of radius  $r = 2500 \text{ km}$ .

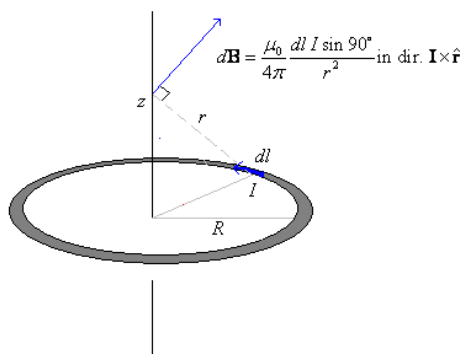


(a) which way would the current have to circulate, as viewed looking down, to create the field in the picture to the left?

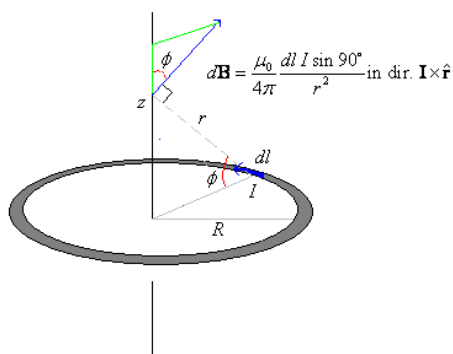
Current would have to circulate clockwise, as viewed from top.

(b) derive an expression for the magnetic field along the central axis of the current loop.

Here's how it goes...



Now we need to break the field into its components. We'll observe that the angle between  $d\mathbf{B}$  and the  $z$ -axis is the same as the angle between the horizontal plane, and  $\mathbf{r}$ .



Only the vertical component of  $d\mathbf{B}$ , namely  $dB_z = dB \cos \phi$  will survive the integration. And so,

$$\begin{aligned}
\mathbf{B} &= \int dB_z \hat{\mathbf{z}} \\
&= \int \frac{\mu_0}{4\pi} \frac{dl I \sin 90^\circ}{r^2} \cos \phi \hat{\mathbf{z}} \\
&= \int \frac{\mu_0}{4\pi} \frac{dl I}{r^2} \cos \phi \hat{\mathbf{z}} \\
&= \frac{\mu_0 I}{4\pi r^2} \cos \phi \int dl \hat{\mathbf{z}} \\
&= \frac{\mu_0 I}{4\pi r^2} \cos \phi (2\pi R) \hat{\mathbf{z}} && \text{integral over the arc is just the arc length} \\
&= \frac{\mu_0 IR}{2r^2} \cos \phi \hat{\mathbf{z}}
\end{aligned}$$

Now

$$\begin{aligned}
r &= \sqrt{z^2 + R^2} \\
\cos \phi &= \frac{R}{\sqrt{z^2 + R^2}}
\end{aligned}$$

Therefore the field is:

$$\mathbf{B} = \frac{\mu_0 IR^2}{2(R^2 + z^2)^{3/2}} \hat{\mathbf{z}}$$

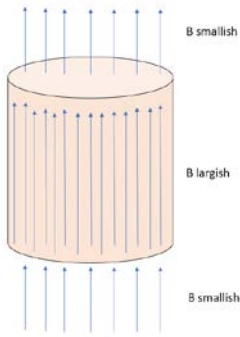
(c) use this expression to estimate the current, if the magnetic field is 50μT at the pole (R = 6400km)

So,

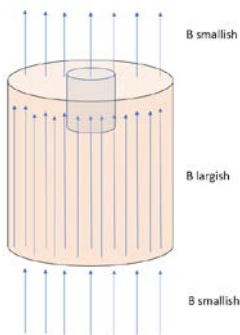
$$\begin{aligned}
50 \times 10^{-6} &= \frac{(4\pi \times 10^{-7}) I (2.5 \times 10^6)^2}{2 \left[ (2.5 \times 10^6)^2 + (6.4 \times 10^6)^2 \right]^{3/2}} \\
50 \times 10^{-6} &= 1.2 \times 10^{-14} I \\
I &= \frac{50 \times 10^{-6}}{1.2 \times 10^{-14}} = 4.2 \times 10^9 \text{ A}
\end{aligned}$$

**Problem 6.** Magnetic materials, like iron, will enhance the magnetic field that permeates them, resulting in the picture below. The picture is roughly true, but not exactly, as it violates Gauss's law. Why? And draw a Gaussian surface to substantiate your argument.





Guassian surface below evinces the problem. The flux going into it surpasses the flux going out, and so the net flux isn't zero, as Gauss's law states it must be.



**Problem 7.** Consider a long hollow cylindrical wire carrying current  $I = 4\text{A}$  to the right, uniformly distributed between the two radii  $R_1 = 2\text{cm}$ , and  $R_2 = 5\text{cm}$ .



(a) The current density  $j$ , is defined to be the current flowing across some cross-section area, divided by that cross-section area. The units of  $j$  are  $\text{A}/\text{m}^2$ . What is the current density here?

Well,

$$j = \frac{I}{A} = \frac{I}{\pi R_2^2 - \pi R_1^2} = \frac{4\text{A}}{\pi(0.05)^2 - \pi(0.02)^2} = 606 \text{ A}/\text{m}^2$$

(b) Use Ampere's law and the inner green Amperian loop to derive a formula for the magnetic field for all radii  $r < R_1$  in terms of  $r$ ,  $\mu_0$ , and numbers.

We got,

$$\oint \mathbf{B} \cdot d\mathbf{r} = \mu_0 I_{\text{enclosed}}$$

$$B \cdot 2\pi r = \mu_0 (0)$$

$$B = 0$$

(c) Use Ampere's law and the mid green Amperian loop to derive a formula for the magnetic field for all radii between  $R_1$  and  $R_2$ , in terms of  $r$ ,  $\mu_0$ , and numbers.

And,

$$\oint \mathbf{B} \cdot d\mathbf{r} = \mu_0 I_{\text{enclosed}}$$

$$B \cdot 2\pi r = \mu_0 j A_{\text{enclosed}}$$

$$B \cdot 2\pi r = \mu_0 (606)(\pi r^2 - \pi \cdot 0.02^2)$$

$$B = \frac{\mu_0 (606)(\pi r^2 - \pi \cdot 0.02^2)}{2\pi r} = \frac{303\mu_0 (r^2 - 0.02^2)}{r}$$

(d) Use Ampere's law and the outer green Amperian loop to derive a formula for the magnetic field for all radii  $r > R_2$ , in terms of  $r$ ,  $\mu_0$ , and numbers.

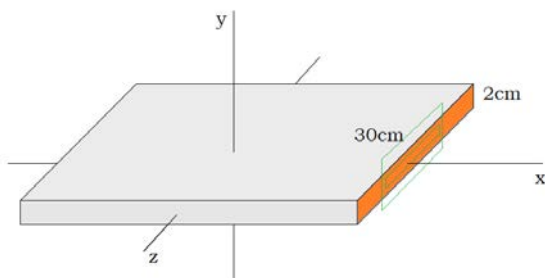
And,

$$\oint \mathbf{B} \cdot d\mathbf{r} = \mu_0 I_{\text{enclosed}}$$

$$B \cdot 2\pi r = \mu_0 (4)$$

$$B = \frac{4\mu_0}{2\pi r} = \frac{2\mu_0}{\pi r}$$

**Problem 8.** Consider a really (fairly) wide sheet with width 30cm, and thickness 2cm. And suppose there is a current  $I = 12\text{A}$  flowing through it from left to right (along the  $+x$  direction). If the sheet is wide enough, then the magnetic field will point purely within the  $x$ - $z$  plane both above and below the sheet.



(a) What is the current density,  $j$ , flowing through the sheet?

That's,

$$j = \frac{I}{A} = \frac{12\text{A}}{(0.02\text{m})(0.30\text{m})} = 2000\text{A/m}^2$$

(b) What, specifically, is the direction of the field for  $y > 0$ . What, specifically, is the direction of the field for  $y < 0$ ?

For  $y > 0$ , it's  $\hat{\mathbf{k}}$ , and for  $y < 0$ , it's  $-\hat{\mathbf{k}}$ .

(c) Use Ampere's law, and the inner Amperian loop, to derive a formula for the magnetic field at all heights  $y$  within the sheet in terms of  $y$ ,  $\mu_0$ , and numbers.

So, let  $\ell$  be the width of the loop, and  $y$  the height. Then,

$$\oint \mathbf{B} \cdot d\mathbf{r} = \mu_0 I_{\text{enclosed}}$$

$$B \cdot 2\ell = \mu_0 j A_{\text{enclosed}}$$

$$B \cdot 2\ell = \mu_0 (2000)(\ell \cdot 2y)$$

$$B = 2000\mu_0 y$$

(d) Use Ampere's law, and the outer Amperian loop to derive a formula for the magnetic field at all heights  $y$  above the sheet in terms of  $y$ ,  $\mu_0$ , and numbers.

So, let  $\ell$  be the width of the loop, and  $y$  the height. Then,

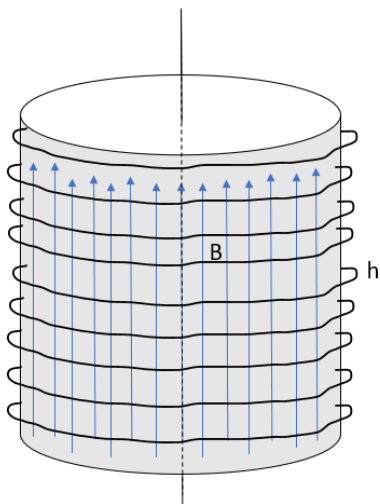
$$\oint \mathbf{B} \cdot d\mathbf{r} = \mu_0 I_{\text{enclosed}}$$

$$B \cdot 2\ell = \mu_0 j A_{\text{enclosed}}$$

$$B \cdot 2\ell = \mu_0 (2000)(\ell \cdot 0.02)$$

$$B = \frac{2000\mu_0(0.02)}{2} = 20\mu_0$$

**Problem 9.** Say you wrap a wire 500 times round an  $h = 20\text{cm}$  tall cylinder. And suppose the wire carries current  $I = 2\text{A}$ .



(a) What is the magnetic field within the cylinder?

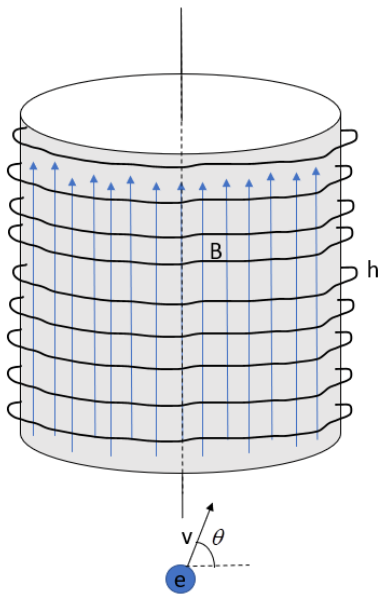
So,

$$\begin{aligned} B &= \mu_0 n I \\ &= (4\pi \times 10^{-7}) \left( \frac{500}{0.20} \right) (2) \\ &= 6.3 \text{ mT} \end{aligned}$$

(b) Which way is the current circulating, as viewed from the top?

Counter-clockwise.

**Problem 10.** Say the magnetic field in the problem above was 5mT, and h was 5m. And an electron enters the solenoid with velocity  $v = 5 \times 10^7 \text{ m/s}$ , at angle  $\theta = 65^\circ$ .



(a) Which way, as viewed from top looking down, will the electron circulate around the field lines? And draw the path as best you can.

Counter-clockwise.

(b) What is the electron's orbital radius?

Well,

$$R = \frac{mv_{\perp}}{qB} = \frac{(9.11 \times 10^{-31} \text{ kg})(5 \times 10^7 \cos 65^\circ \text{ m/s})}{(1.6 \times 10^{-19} \text{ C})(5 \times 10^{-3} \text{ T})} = 2.4 \text{ cm}$$

(c) What will be the orbital period of the electron?

Period is:

$$T = \frac{2\pi R}{v_{\perp}} = \frac{2\pi(0.024\text{m})}{5 \times 10^7 \cos 65^\circ} = 7.1\text{ns}$$

(d) How long until the electron exits the solenoid?

This is:

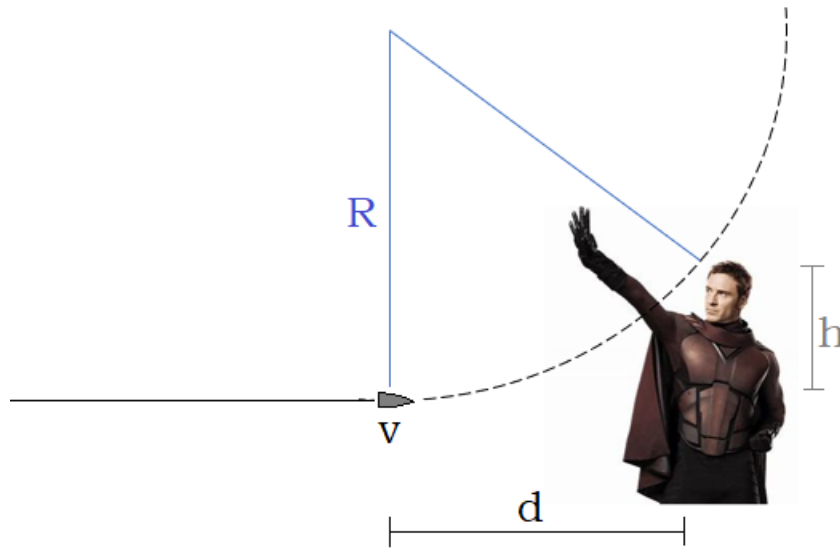
$$\Delta t = \frac{h}{v_{\parallel}} = \frac{5\text{m}}{5 \times 10^7 \sin 65^\circ} = 110\text{ns}$$

(e) How many loops will it complete before it exits?

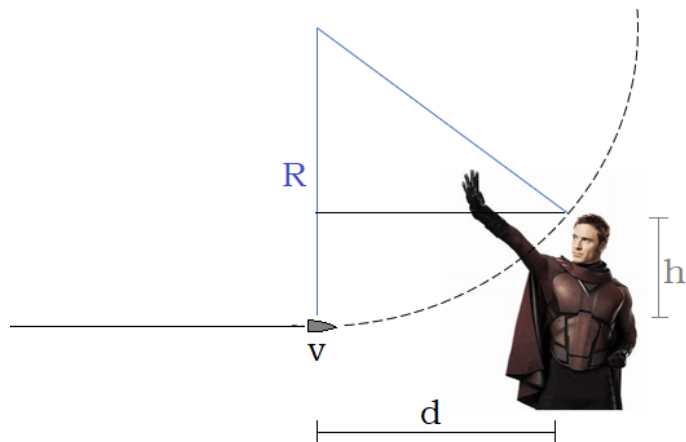
Well,

$$\# \text{ loops} = \frac{\Delta t}{T} = \frac{110\text{ns}}{7.1\text{ns}} = 15.5$$

**Problem 11.** Suppose you're Magneto and you create a magnetic field to deflect a bullet traveling with speed  $v = 500\text{m/s}$ , and charge  $q = 2\text{pC}$ . Give the magnitude and direction of the magnetic field you must create if you must deflect the bullet by  $h = 40\text{cm}$ , when it's  $d = 50\text{m}$  away? Let mass of bullet be  $m = 10\text{g}$ . And I've drawn the radius of the circle it'll partially execute, for your convenience.



Field must point into the page, if the force is to be upwards, towards the center. And then we have, considering that triangle,



$$R^2 = d^2 + (R - h)^2$$

$$R^2 = d^2 + R^2 - 2hR + h^2$$

$$2hR = d^2 + h^2$$

$$R = \frac{d^2 + h^2}{2h} = \frac{50^2 + (0.40)^2}{2(0.40)} = 3120 \text{ m}$$

Also,

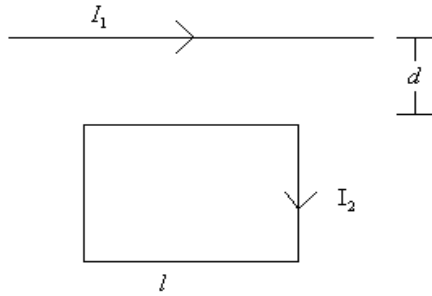
$$R = \frac{mv_{\perp}}{qB}$$

and so we have:

$$3120 \text{ m} = \frac{(0.010 \text{ kg})(500 \text{ m/s})}{(2 \times 10^{-12} \text{ C})B}$$

$$B = \frac{(0.010 \text{ kg})(500 \text{ m/s})}{(2 \times 10^{-12} \text{ C})(3120 \text{ m})} = 8 \times 10^8 \text{ T}$$

**Problem 11.** Consider the two wires below. What is the net force the long straight wire exerts on the square current loop (with side length  $\ell$ ), and in what direction does it point? Let  $I_1 = 4\text{A}$ ,  $I_2 = 6\text{A}$ ,  $d = 10\text{cm}$ , and  $\ell = 5\text{cm}$ .



First we need the magnetic field through the two wires. The field at the top wire is:

$$\begin{aligned}\mathbf{B}_{top} &= \frac{\mu_0 I}{2\pi r} \text{ in dir. } \mathbf{I} \times \mathbf{r} \\ &= \frac{(4\pi \times 10^{-7})(4)}{2\pi(0.1)} (-\hat{\mathbf{k}}) \\ &= 8 \times 10^{-6} (-\hat{\mathbf{k}})\end{aligned}$$

and on bottom,

$$\begin{aligned}\mathbf{B}_{bot} &= \frac{\mu_0 I}{2\pi r} \text{ in dir. } \mathbf{I} \times \mathbf{r} \\ &= \frac{(4\pi \times 10^{-7})(4)}{2\pi(0.15)} (-\hat{\mathbf{k}}) \\ &= 0.53 \times 10^{-5} (-\hat{\mathbf{k}})\end{aligned}$$

The force on the top wire is:

$$\begin{aligned}\mathbf{F}_{top} &= lIB \sin \theta \text{ in dir. } \mathbf{I} \times \mathbf{B} \\ &= (0.05)(6)(8 \times 10^{-6}) \hat{\mathbf{k}} \\ &= 2.4 \times 10^{-6} \hat{\mathbf{k}}\end{aligned}$$

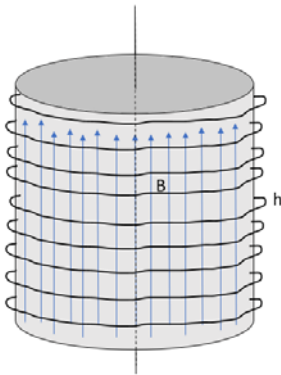
and on bottom,

$$\begin{aligned}\mathbf{F}_{bot} &= lIB \sin \theta \text{ in dir. } \mathbf{I} \times \mathbf{B} \\ &= (0.05)(6)(5.3 \times 10^{-5}) (-\hat{\mathbf{k}}) \\ &= -1.6 \times 10^{-6} \hat{\mathbf{k}}\end{aligned}$$

So the net force is:

$$\mathbf{F} = 8 \times 10^{-7} \text{ N } \hat{\mathbf{k}}$$

**Problem 12.** Going back to the solenoid in a previous problem, let's say the wire is wrapped 500 times round the  $h = 20\text{cm}$  tall cylinder, with diameter  $d = 10\text{cm}$ , and carries a  $2\text{A}$  current.



(a) What is the magnetic potential energy in the solenoid?

This is:

$$\begin{aligned}
 PE_B &= \frac{1}{2\mu_0} \int B^2 dV \\
 &= \frac{1}{2\mu_0} B^2 \cdot \text{Volume} && B \text{ is constant so it can come outside the integral} \\
 &= \frac{1}{2\mu_0} B^2 \pi r^2 h \\
 &= \frac{1}{2\mu_0} (6.3 \times 10^{-3})^2 \pi (0.05)^2 (0.20) \\
 &= 2.5\text{mJ}
 \end{aligned}$$

(b) Now say we have an iron core with the same dimensions as the solenoid, and with magnetic susceptibility  $\chi_m = (\text{party like its}) 1999$ . Will the core be attracted to the solenoid, or repelled?

Attracted, cause scientific reasons.

(c) Now its placed inside the solenoid. What is the magnetic field inside the solenoid now?

The old field was  $6.3\text{mT}$ . And so the new field, with the iron core, will be  $B = (1 + \chi_m)B_0 = (2000)(6.3\text{mT}) = 12.6\text{T}$ .

(d) What is the magnetic potential energy in the solenoid now?

That's,



$$\begin{aligned}
 PE_B &= \frac{1}{2\kappa_m\mu_0} \int B^2 dV \\
 &= \frac{1}{\kappa_m} PE_B \quad B \text{ is constant so it can come outside the integral} \\
 &= \frac{1}{2000} 99\text{kJ} \\
 &= 50\text{J}
 \end{aligned}$$

(e) Heck, why stop now. Let's say we try to insert a perfect diamagnet, like a superconductor, with the same dimensions as the iron core, and susceptibility  $\chi_m = -1$ . Would it be attracted to or repelled by the solenoid?

Repelled, because of other scientific reasons.

(f) Now say we place it inside the solenoid, what would be the magnetic field inside the solenoid now? The old field was 6.3mT. And so the new field, with the iron core, will be  $B = (1-\chi_m)B_0 = (0)(6.3\text{mT}) = 0\text{T}$ .

(g) What is the magnetic potential energy in the solenoid now?

So,

$$\begin{aligned}
 PE_B &= \frac{1}{2\kappa_m\mu_0} \int B^2 dV \\
 &= \frac{1}{2(1+\chi_m)\mu_0} \int (1+\chi_m)^2 B_0^2 dV \\
 &= \frac{1+\chi_m}{2\mu_0} \int B_0^2 dV \\
 &= \frac{1+(-1)}{2\mu_0} \int B_0^2 dV \\
 &= 0
 \end{aligned}$$